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ASYMPTOTIC METHOD OF SOLUTION OF THE FROZEN DISSOCIATED LAMINAR BOUNDARY-LAYER FLOW OVER A FLAT-PLATE SURFACE WITH ARBITRARILY DISTRIBUTED CATALYCITY*

T. Y. LI† and P. S. KIRK‡

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Abstract—Meksyn's method [1] of solution of laminar boundary layers is extended to deal with a typical problem of a binary boundary layer with nonsimilar solution. The present paper deals with the asymptotic method of solution of the frozen dissociated laminar boundary-layer flow over a flat-plate surface with arbitrarily distributed catalycity. A closed form solution is obtained and has been compared with existing data due to other investigators [2–5]. Results of the comparison show that the present results are of adequate accuracy. The attractiveness of the present method is in the simplicity of the analysis.

NOMENCLATURE

- A_m, coefficients defined by equation (21);
- a_n, coefficients defined by equation (12);
- B_λ, constant in equation (39);
- $c_m^{(1)}$, coefficients defined by equation (21a);
- f, Blasius function;
- K_1, molecular mass function;
- k_n, coefficients defined by equation (16);
- S, Schmidt number $(= \mu/\rho D_{12})$;
- u, v, velocities in x- and y-direction;
- x, y, boundary-layer coordinates;
- z_0, surface atom mass function.

Greek symbols

- β, defined by equation (43);
- γ, defined by equation (42);
- ε, dummy variable in equation (29);
- τ_1, defined by equation (29);
- τ, variable defined by equation (10);
- ξ, η, transformed coordinates;
- ρ, density;
- μ, viscosity;
- λ, exponent in catalycity variation function;
- $\bar{\lambda}$, density viscosity ratio $(= \rho\mu/\rho_e\mu_e)$;
- φ, Dahmkohler number;
- ϕ, function defined by equation (11);
- σ, variable defined by equation (21a);
- ψ, defined by equation (45);
- χ, defined by equation (46).

Subscripts

- 1, refers to molecular species;
- 2, refers to atomic species;
- e, evaluated in free-stream conditions;
- 0, evaluated for $\eta = y = 0$, i.e. at the surface.

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† Ohio State University, Columbus, Ohio.
‡ Boeing Aircraft Company, Seattle, Washington.

INTRODUCTION

THE ASYMPTOTIC method of solution of laminar boundary layers, first used by Meksyn [1], has been applied to the study of the boundary-layer diffusion equation in the presence of a body surface with first order atom recombination rate distributed in an arbitrary continuous manner. In the present paper, we deal with the casé of a frozen dissociated laminar boundary-layer flow over a flat-plate surface with arbitrarily distributed catalycity, constancy of viscosity-density product being assumed. When the nondimensional Dahmkohler number varies as a single power of the distance [see equation (39)], we obtain the surface atomic mass fraction on the flat plate in a closed form solution [see equation (51)]. Our approximate results are compared with the "exact" solution by Inger [2] and Chambré–Acrivos [3]; the solution by integral method by Chung–Anderson [4]; and the solution by Lighthill's technique due to Chung–Liu–Mirels [5]*. It will be seen that the asymptotic method of solution presented herein provides adequate accuracy. The attractiveness of the present method is mainly in the simplicity of the approach. Previously, the present authors have extended Meksyn's method to obtain some similar solutions of binary boundary layers [6] in connection with the transpiration cooling problem, and had reasonably encouraging results. The present study suggests that the extended Meksyn's method for the treatment of nonsimilar solutions of the binary boundary-layer equations deserves further attention. A general formulation of this latter problem will be presented elsewhere.

ANALYSIS

Consider the laminar boundary-layer flow of a dissociated binary gas mixture over a flat plate. Let there be an arbitrarily prescribed surface catalycity. We assume that the mixture of molecules and atoms is chemically frozen in the boundary layer. We further assume that the function representing the velocity profile is self-similar and independent of the energy and diffusion equations. In other words, the velocity profile is expressible as follows:

$$\frac{u}{u_e} = f(\eta) \tag{1}$$

* Some of these results were compared with the results obtained by the method of Mellin transform by Simpkins and Freeman [7, 8].

where η is the similarity variable defined as follows:

$$\eta = \frac{\rho_e u_e}{\sqrt{(2\xi)}} \int_0^y \frac{\rho}{\rho_e} \, \mathrm{d}y \tag{2}$$

and ξ is defined as

$$\xi = \int_0^x \rho_e u_e \mu_e \, \mathrm{d}x. \tag{3}$$

The differential equation for f is

$$(\lambda f'')' + ff'' = 0 \tag{4}$$

with the boundary conditions:

$$f(0) = f'(0) = 0 \qquad f'(\infty) = 1. \tag{5}$$

In equation (4), $\lambda = \rho\mu/\rho_e\mu_e$. We shall take $\lambda = 1$, and f is then the Blasius function. In order to determine the surface atom recombination rate resulting from the prescribed surface catalycity, we must consider the nonsimilar solution of the diffusion equation:

$$K_1'' + SfK_1' = 2\xi f'S \frac{\partial K_1}{\partial \xi}. \tag{6}$$

The boundary conditions are:

$$K_1(\infty, \xi) = 0 \tag{7}$$

$$\frac{\partial K_1}{\partial \eta}(0, \xi) = -\varphi(\xi)\left[1 - K_1(0, \xi)\right]. \tag{8}$$

In these equations $K_1(\xi, \eta)$ denotes the mass fraction of the molecular species. Equation (8) is the relation between the diffusion of species at the surface and the surface recombination rate. $\varphi(\xi)$ is termed the Dahmkoler number and represents the ratio of atom diffusion time to surface recombination time. It is a prescribed function of ξ. The present objective is to determine $K_1(0, \xi)$.

Integrating equation (6), we have

$$\frac{\partial K_1}{\partial \eta} = \mathrm{e}^{-\tau}\phi(\xi, \eta) \tag{9}$$

where

$$\tau = S \int_0^\eta f \, \mathrm{d}\eta \tag{10}$$

$$\phi(\xi, \eta) = \frac{\partial K_1}{\partial \eta}(0, \xi) + \int_0^\eta e^\tau 2\xi f' S \frac{\partial K_1}{\partial \xi} d\eta. \quad (11)$$

S has been taken to be constant. The velocity profile is expressible as

$$f = \sum_{n=0}^{\infty} \frac{a_n}{n!} \eta^n \quad (12)$$

where $a_0 = a_1 = a_3 = a_4 = 0$ and a_n are independent of ξ. From equation (10), we have

$$\tau = S \sum_{n=0}^{\infty} \frac{a_n}{(n+1)!} \eta^{n+1}. \quad (13)$$

written in the form:

$$\tau^n = (S\eta)^n \sum_{l=0}^{\infty} S_l^{(n)} \eta^l$$

where

$$S_l^{(n)} = \sum_{m=0}^{l} S_m^{(1)} S_{l-m}^{(n-1)}. \quad (13b)$$

Consequently each coefficient may be expressed in terms of "lower order" coefficients, and eventually in terms of $S_n^{(0)}$ and $S_n^{(1)}$ which are the basic coefficients in the series τ. Thus, we may write:

$$(\tau)^0 = (S\eta)^0 \sum_{n=0}^{\infty} S_n^{(0)} \eta^n = 1 \qquad S_n^{(0)} = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$(\tau)^1 = (S\eta)^1 \sum_{n=0}^{\infty} S_n^{(1)} \eta^n \qquad S_n^{(1)} = \frac{a_n}{(n+1)!}.$$

$$(\tau)^2 = (S\eta)^2 \sum_{n=0}^{\infty} S_n^{(2)} \eta^n \qquad S_n^{(2)} = \sum_{m=0}^{n} S_m^{(1)} S_{n-m}^{(1)}$$

$$\cdots \qquad \cdots$$

$$(\tau)^n = (S\eta)^n \sum_{l=0}^{\infty} S_l^{(n)} \eta^l \qquad S_l^{(n)} = \sum_{m=0}^{l} S_m^{(1)} S_{l-m}^{(n-1)}.$$

For further work it will be necessary to expand the function τ^n as a series of the variable η.

Consider

$$\tau^n = (S\eta)^n \left\{ \sum_{k=0}^{\infty} \frac{a_k}{(k+1)!} \eta^k \right\}^n. \quad (13a)$$

If we now define $S_l^{(n)}$ to be the coefficient of η^l in the expansion of the function

$$\left\{ \sum_{k=0}^{\infty} \frac{a_k}{(k+1)!} \eta^k \right\}^n,$$

it is then clear that the expansion may be

Then, we have

$$e^\tau = \sum_{n=0}^{\infty} \frac{1}{n!} (\tau)^n = \sum_{n=0}^{\infty} W_n \eta^n \quad (14)$$

where, utilizing the result (13b),

$$W_n = \sum_{p=0}^{n} (S)^p S_{n-p}^{(p)} \frac{1}{p!}. \quad (15)$$

From equation (15), we obtain

$$W_0 = 1$$

$$W_1 = S\, S_0^{(1)}$$

$$W_2 = S\,S_1^{(1)} + \tfrac{1}{2}S^2 S_0^{(2)}$$

$$W_3 = S\,S_2^{(1)} + \tfrac{1}{2}S^2 S_1^{(2)} + \frac{1}{3!}S^3 S_0^{(3)}$$

$$\cdots\cdots$$

We now expand the nonsimilar solution for K_1 as follows:

$$K_1(\eta, \xi) = \sum_{n=0}^{\infty} \frac{k_n(\xi)}{n!}\eta^n. \tag{16}$$

By equations (12) and (16), we obtain

$$f'\frac{\partial K_1}{\partial \xi} = \sum_{n=0}^{\infty} B_n \eta^n \tag{17}$$

where

$$B_n = \sum_{m=0}^{n} \frac{a_{m+1}}{m!}\frac{k'_{n-m}}{(n-m)!}. \tag{18}$$

Therefore, we have (for $a_0 = a_1 = a_3 = a_4 = 0$):

$$B_0 = 0$$

$$B_1 = a_2 k'_0$$

$$B_2 = a_2 k'_1$$

$$B_3 = \tfrac{1}{2}a_2 k'_2$$

$$\cdots\cdots$$

Combining equations (11), (14), and (17) we obtain

$$\phi(\xi, \eta) = \sum_{m=0}^{\infty} p_m \eta^m \tag{19}$$

where

$$p_0 = k_1(\xi)$$

$$p_1 = 0$$

$$p_2 = S\xi(a_2 k'_0)$$

$$p_3 = \tfrac{2}{3}S\xi(a_2 k'_1 + SS_0^{(1)} a_2 k'_0)$$

$$p_4 = \tfrac{1}{2}S\xi[\tfrac{1}{2}a_2 k'_2 + SS_0^{(1)} a_2 k'_1 + (SS_1^{(1)} + S^2 S_0^{(2)}\tfrac{1}{2})a_2 k'_0]$$

$$\cdots\cdots$$

Substituting equation (19) into equation (9) and integrating again, we obtain

$$K_1(\eta, \xi) - k_0(\xi) = \int_0^{\eta} \Big(\sum_{m=0}^{\infty} p_m \eta^m\Big)\, \mathrm{e}^{-\tau} \mathrm{d}\eta. \tag{20}$$

By inversion of the series in equation (13), we have [1]

$$\eta = \sum_{m=0}^{\infty} \frac{A_m}{m+1}\left(\frac{\tau}{S}\right)^{\frac{1}{3}(m+1)} \tag{21}$$

where

$$A_0 = \left(\frac{a_2}{6}\right)^{-\frac{1}{3}}$$

$$A_1 = A_2 = 0$$

$$A_3 = \tfrac{1}{15}\left(\frac{a_2}{6}\right)^{-\frac{4}{3}}$$

$$\cdots\cdots$$

It follows that

$$\frac{\mathrm{d}\eta}{\mathrm{d}\tau} = \frac{1}{3\tau}\sum_{m=0}^{\infty} A_m\left(\frac{\tau}{S}\right)^{\frac{1}{3}(m+1)} \tag{22}$$

Equation (21) may be written as:

$$\eta = \sigma \sum_{m=0}^{\infty} c_m^{(1)} \sigma^m$$

where

$$c_m^{(1)} = \frac{A_m}{m+1}$$

and (21a)

$$\sigma = \left(\frac{\tau}{S}\right)^{\frac{1}{3}}.$$

We obtain then

$$\eta^2 = \sigma^2 \sum_{m=0}^{\infty} c_m^{(2)} \sigma^m \qquad c_m^{(2)} = \sum_{l=0}^{m} c_l^{(1)} c_{m-l}^{(1)}$$

$$\eta^3 = \sigma^3 \sum_{m=0}^{\infty} c_m^{(3)} \sigma^m \qquad c_m^{(3)} = \sum_{l=0}^{m} c_l^{(1)} c_{m-l}^{(2)}$$

$$\cdots \qquad \cdots$$

$$\eta^p = \sigma^p \sum_{m=0}^{\infty} c_m^{(p)} \sigma^m \qquad c_m^{(p)} = \sum_{l=0}^{m} c_l^{(1)} c_{m-l}^{(p-1)}.$$

It follows that

$$\sum_{m=0}^{\infty} p_m \eta^m = \sum_{m=0}^{\infty} l_m (\tau/S)^{m/3} \tag{23}$$

where

$$l_m = \sum_{n=0}^{m} p_n c_{m-n}^{(n)} \tag{24}$$

we have defined here

$$c_l^{(0)} = \begin{cases} 1 & l = 0 \\ 0 & l \neq 0. \end{cases}$$

From equation (24), we obtain

$$l_0 = p_0 = k_1(\xi)$$

$$l_1 = p_1 c_0^{(1)} = 0$$

$$l_2 = p_1 c_1^{(1)} + p_2 c_0^{(2)} = S\xi(a_2 k_0')\left(\frac{a_2}{6}\right)^{-\frac{2}{3}}$$

$$l_3 = p_1 c_2^{(1)} + p_2 c_1^{(2)} + p_3 c_0^{(3)} = 4S\xi k_1'$$

$$\cdots\cdots$$

Combining equations (20), (22), and (23) we obtain

$$K_1(\eta, \xi) - k_0 = \sum_{n=0}^{\infty} \tfrac{1}{3}(S)^{-(n+1)/3} m_n \times \int_0^{\tau} {}^{[(n+1)/3]-1}\, e^{-\tau}\, d\tau \tag{25}$$

where

$$m_n = \sum_{m=0}^{n} l_m A_{n-m}. \tag{26}$$

Thus we have

$$m_0 = l_0 A_0 = k_1 \left(\frac{a_2}{6}\right)^{-\frac{1}{3}}$$

$$m_1 = l_0 A_1 + l_1 A_0 = 0$$

$$m_2 = l_0 A_2 + l_1 A_1 + l_2 A_0 = 6S\xi k_0'$$

$$m_3 = l_0 A_3 + l_1 A_2 + l_2 A_1 + l_3 A_0 = \left(\frac{k_1}{15} + 4S\xi k_1'\right)\left(\frac{a_2}{6}\right)^{-\frac{1}{3}}$$

$$\cdots\cdots$$

Combining equations (7) and (25), we obtain

$$k_0 + \tfrac{1}{3} \sum_{n=0}^{\infty} (S)^{-(n+1)/3} m_n \times \int_0^{\infty} e^{-\tau} \tau^{[(n+1)/3]-1} d\tau = 0 \tag{27}$$

whence we have

$$k_0 + \tfrac{1}{3} \sum_{n=0}^{\infty} (S)^{-(n+1)/3} m_n \Gamma[(n+1)/3] = 0. \tag{28}$$

To improve the convergence of the above series, Euler transformation may be applied. For this purpose, we introduce a parameter ε as follows:

$$\tau = \tau_1 \varepsilon^3. \tag{29}$$

Thus equation (27) can be rewritten as

$$k_0 + \tfrac{1}{3} \sum_{n=0}^{\infty} (S)^{-(n+1)/3} \times m_n \left\{ \int_0^{\infty} e^{-\tau_1 \varepsilon^3} \tau_1^{(n-2)/3} d\tau_1 \right\} \varepsilon^{n+1} = 0. \tag{30}$$

Now

$$\lim_{\varepsilon \to 1} \left\{ \int_0^{\infty} e^{-\tau_1 \varepsilon^3} \tau_1^{(n-2)/3}\, d\tau_1 \right\} = \Gamma[(n+1)/3].$$

Let

$$g_n = \tfrac{1}{3}(S)^{-(n+1)/3} m_n \Gamma[(n+1)/3] \tag{31}$$

then equation (30) becomes

$$k_0 + \sum_{n=0}^{\infty} g_n \varepsilon^{n+1} = 0. \tag{32}$$

By Euler transformation, we have

$$\sum_{n=0}^{\infty} g_n \varepsilon^{n+1} = \sum_{n=0}^{\infty} b_n \left(\frac{\varepsilon}{1+\varepsilon}\right)^{n+1} \tag{33}$$

where

$$b_n = \sum_{l=0}^{n} \frac{n!}{l!(n-l)!} g_l. \tag{34}$$

Combining equations (32) and (33) we obtain for $\varepsilon \to 1$

$$k_0 + \sum_{n=0}^{\infty} (\tfrac{1}{2})^{n+1} b_n = 0 \tag{35}$$

where

$$b_0 = g_0 = \tfrac{1}{3}\,\Gamma(\tfrac{1}{3})\,k_1\left(\frac{a_2 S}{6}\right)^{-\frac{1}{3}}$$

$$b_1 = g_0 + g_1 = \tfrac{1}{3}\,\Gamma(\tfrac{1}{3})\,k_1\left(\frac{a_2 S}{6}\right)^{-\frac{1}{3}}$$

$$b_2 = g_0 + 2g_1 + g_2 = \tfrac{1}{3}\,\Gamma(\tfrac{1}{3})\,k_1\left(\frac{a_2 S}{6}\right)^{-\frac{1}{3}} + 2\xi k_0'$$

$$\begin{aligned} b_3 &= g_0 + 3g_1 + 3g_2 + g_3 \\ &= \tfrac{1}{3}\,\Gamma(\tfrac{1}{3})\,k_1\left(\frac{a_2 S}{6}\right)^{-\frac{1}{3}} + 6\xi\,k_0' \\ &\quad + \tfrac{1}{3}\,\Gamma(\tfrac{4}{3})\,S^{-1}\left(\frac{a_2 S}{6}\right)^{-\frac{1}{3}}\left(\frac{k_1}{15} + 4S\xi\,k_1'\right). \end{aligned}$$

Substitution of these results into equation (35) yields

$$\begin{aligned} k_0 &+ \tfrac{15}{48}\,\Gamma(\tfrac{1}{3})\left(\frac{a_2 S}{6}\right)^{-\frac{1}{3}} k_1 + \tfrac{5}{8}\xi\,k_0' \\ &+ \tfrac{1}{48}\,\Gamma(\tfrac{4}{3})\,S^{-1}\left(\frac{a_2 S}{6}\right)^{-\frac{1}{3}}\left(\frac{k_1}{15} + 4S\xi\,k_1'\right) \\ &+ \ldots = 0. \end{aligned} \tag{36}$$

From equation (8), we have

$$k_1(\xi) = -\,\varphi(\xi)\,[1 - k_0(\xi)]. \tag{37}$$

Therefore,

$$k_1' = -\,\varphi'(1 - k_0) + \varphi k_0'. \tag{38}$$

The prescribed surface catalycity is assumed in the following form:

$$\varphi = B_\lambda \xi^\lambda \tag{39}$$

then

$$\left.\begin{aligned} \xi\varphi' &= \lambda\varphi \\ \xi k_0' &= \lambda\varphi\frac{\mathrm{d}k_0}{\mathrm{d}\varphi}. \end{aligned}\right\} \tag{40}$$

These equations may be combined with equation (36) to yield

$$\frac{\mathrm{d}k_0}{\mathrm{d}\varphi} + \left(\frac{1}{\gamma\varphi} + \frac{\beta}{\gamma}\right)k_0 = \frac{\beta}{\gamma} \tag{41}$$

where

$$\gamma = \left[\tfrac{5}{8} + \tfrac{1}{12}\Gamma(\tfrac{4}{3})\left(\frac{a_2 S}{6}\right)^{-\frac{1}{3}}\varphi\right]\lambda \tag{42}$$

$$\begin{aligned} \beta = \left(\frac{a_2 S}{6}\right)^{-\frac{1}{3}}&[\tfrac{15}{48}\Gamma(\tfrac{1}{3}) + \tfrac{1}{15}(\tfrac{1}{48})\Gamma(\tfrac{4}{3})S^{-1} \\ &+ \tfrac{1}{12}\Gamma(\tfrac{4}{3})\lambda]. \end{aligned} \tag{43}$$

Determination of $k_1(0,\ \xi)$, i.e. $k_0(\xi)$ can be carried out by the solution of equation (41).

Equation (41) is a linear equation for k_0. It can be integrated by the usual procedure. However, we shall follow a new scheme wherein a simple method of truncation is used. Thus, equation (41) can be written as

$$\frac{\mathrm{d}k_0}{\mathrm{d}\varphi} + \psi k_0 = \chi \tag{44}$$

where

$$\psi = \frac{1}{\gamma\varphi}(1 + \beta\varphi) \tag{45}$$

$$\chi = \frac{\beta}{\gamma}. \tag{46}$$

By differentiation of equation (44), we have

$$\frac{\mathrm{d}^2 k_0}{\mathrm{d}\varphi^2} + \frac{\mathrm{d}\psi}{\mathrm{d}\varphi}k_0 + \psi\frac{\mathrm{d}k_0}{\mathrm{d}\varphi} = \frac{\mathrm{d}\chi}{\mathrm{d}\varphi}. \tag{47}$$

Equations (44) and (47) contain three unknowns, k_0, $\mathrm{d}k_0/\mathrm{d}\varphi$, and $\mathrm{d}^2k_0/\mathrm{d}\varphi^2$. Again, equation (47) may be differentiated, the resulting equation can be added to the system of equations (44) and (47) which then becomes a system of three equations with four unknowns.

This operation may continue indefinitely, unless we truncate the operation at some stage and assume the higher derivatives become negligible. In the present calculation, we assume $\mathrm{d}^2k_0/\mathrm{d}\varphi^2$ is negligible and equation (47) then becomes

$$\frac{\mathrm{d}\psi}{\mathrm{d}\varphi}k_0 + \psi\frac{\mathrm{d}k_0}{\mathrm{d}\varphi} = \frac{\mathrm{d}\chi}{\mathrm{d}\varphi}. \tag{47a}$$

Solving the system of equations (44) and (47a), we have

$$\frac{dk_0}{d\varphi} = \frac{\dfrac{d\chi}{d\varphi} - \dfrac{\chi}{\psi}\dfrac{d\psi}{d\varphi}}{\psi - \dfrac{1}{\psi}\dfrac{d\psi}{d\varphi}} \tag{48}$$

$$k_0 = \frac{\chi}{\psi} - \frac{1}{\psi}\left\{\frac{\dfrac{d\chi}{d\varphi} - \dfrac{\chi}{\psi}\dfrac{d\psi}{d\varphi}}{\psi - \dfrac{1}{\psi}\dfrac{d\psi}{d\varphi}}\right\}. \tag{49}$$

Combining equations (45), (46), and (49) we obtain

$$k_0 = \frac{1}{1+\beta\varphi}\left[\beta\varphi - \frac{\beta\gamma\varphi}{(1+\beta\varphi)^2 + \gamma + \varphi(1+\beta\varphi)\, d\gamma/d\varphi}\right]. \tag{50}$$

Equation (50) is therefore the approximate solution of the present problem. In our numerical examples, we shall deal with the surface atomic mass fraction defined as

$$Z_0 = 1 - k_0 = \frac{1}{1+\beta\varphi} \times \left[1 + \frac{\beta\gamma\varphi}{(1+\beta\varphi)^2 + \gamma + \varphi(1+\beta\varphi)\, d\gamma/d\varphi}\right]. \tag{51}$$

NUMERICAL RESULTS

Figure 1 shows the variation of atom concentration along a flat plate taking $\lambda = 0{\cdot}5$. Results of 6 different solutions are shown on the graph contrasting the relative accuracies. It may be seen that the solution utilizing the assumption of local similarity is at variance with the other data. One would therefore deduce that such an assumption is not valid in this problem, as was anticipated. The other approximate solutions are shown, the first being an integral method by Chung and Anderson [4]. This method appears to be fairly complicated and involves numerical integration of a first order differential equation for the wall mass fraction. It also requires that the pressure gradient term in the momentum equation is neglected. For the case considered, the results from this method are almost identical with those of the present analysis.

The solution by G. Inger [2] represents a series solution of the diffusion equation similar to that of the present analysis, the series being expressed as a power series in the streamwise coordinate. Here it appears that a very large number of terms in the series must be taken, and for 10 terms the solution still diverges at very modest values of the streamwise coordinate.

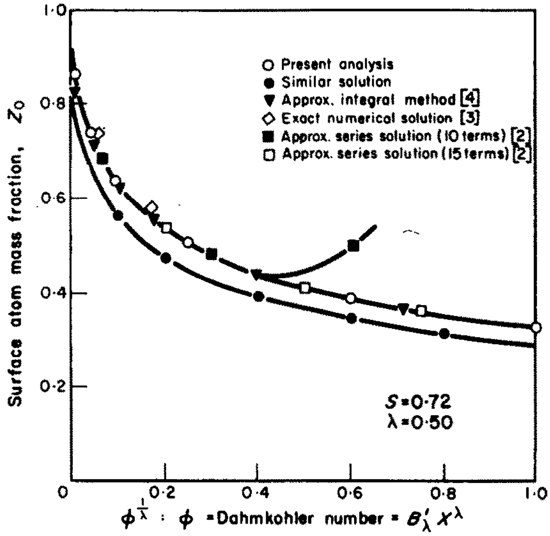

FIG. 1. Variation of atom fraction along a flat plate.

Taking 15 terms in the series, the results showed no divergence in the range indicated, but no doubt still diverge outside this range. However, within the limits of convergence the results are very close to the present analysis.

An exact solution by Chambré and Acrivos [3] was also available for this particular situation, and agrees very closely with the present results.

Figure 2 again represents results for flow along a flat plate, but for this case $\lambda = 1{\cdot}0$. It can be seen that the local similarity solution shows greater deviation from the other solutions than in the previous case. Results from the present analysis agree very closely with those from the Lighthill–Volterra method [5], but differ slightly at high values of φ from the results of Inger which diverge considerably in this region.

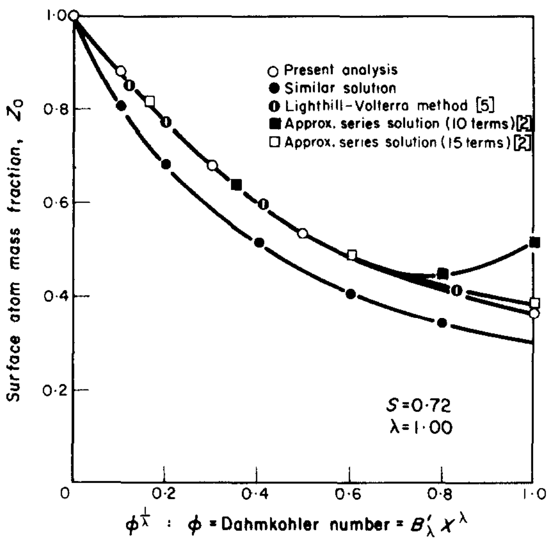

FIG. 2. Variation of atom fraction along a flat plate.

Figure 3 shows results for the case when $\lambda = 1{\cdot}5$. The local similarity solution shows greater divergence from the other results. The data from the Lighthill–Volterra method [5] is seen to be in very good agreement with the present analysis, while those of Inger now show discrepancies at very small values of φ. However, at large values, the results remain fairly close, except where only 10 terms were used in the series.

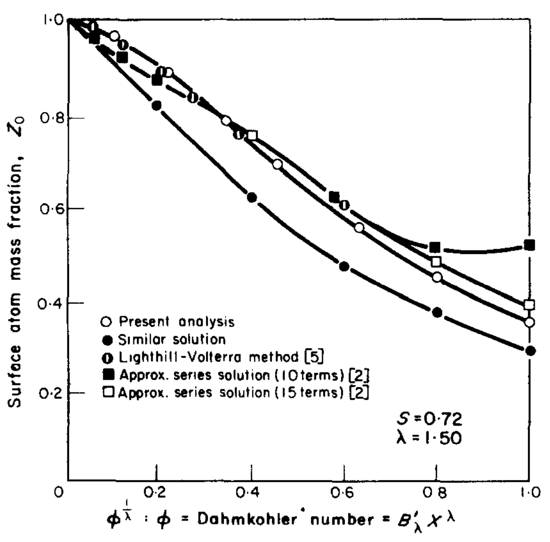

FIG. 3. Variation of atom fraction along a flat plate.

Figure 4 shows that the results of Inger and those of the present analysis are in reasonably good agreement for $\lambda = 2{\cdot}0$. The local similarity solution now shows appreciable deviation from these two solutions.

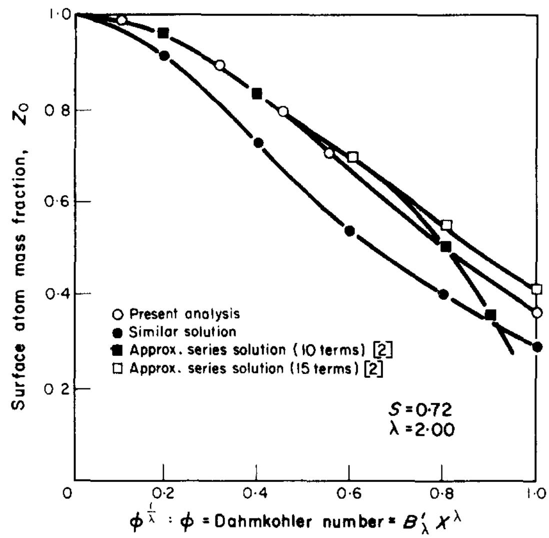

FIG. 4. Variation of atom fraction along a flat plate.

Figures 5 and 6 show the variation of the atom mass fraction along a flat plate for the cases $\lambda = 1{\cdot}0$ and $\lambda = 2{\cdot}0$. However, the scales are plotted logarithmically so as to display the results at high values of φ. It is clear that the results of Inger (15 term solution) and the present analysis are in close agreement throughout the entire range, and both tend toward the local similarity solution.

It is of interest to note that in deriving equation (50) we made the assumption that the terms involving $d^2k_0/d\varphi^2$ were negligible. The results would indicate that this is reasonably true in the cases taken. Although $d^2k_0/d\xi^2$ becomes quite large for small values of λ, the

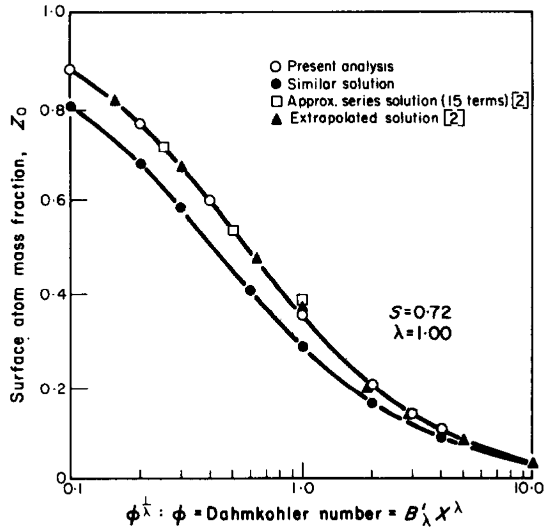

FIG. 5. Variation of atom fraction along a flat plate.

term $d^2k_0/d\varphi^2$ remains small and may be neglected. It is therefore assumed that expressing the solution as a function of φ, i.e. regarding φ as the independent variable, has eliminated problems of convergence which may have been encountered had we attempted to solve directly in terms of the ξ variable.

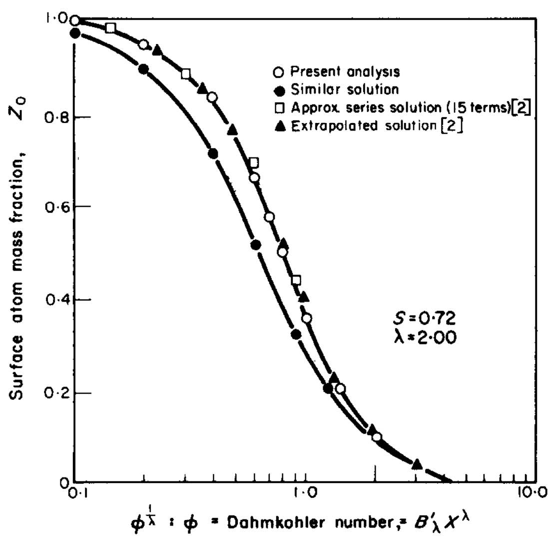

FIG. 6. Variation of atom fraction along a flat plate.

CONCLUSIONS

The results have indicated the reliability of the present solution at least for the case of flat-plate flows. The simplicity of the solution both in analysis and final result gives the approach a great advantage over more complex techniques. The present theory demonstrates the application of Meksyn's method to a boundary-layer problem with nonsimilar solution. The range of this type of solution is very great, it is not necessary to assume that the momentum equation conforms to the laws of similarity, nor that the three basic equations of momentum, energy, and diffusion are independent. It is possible to extend the analysis to provide a complete solution to the entire problem of the nonsimilar boundary-layer equations including such features as injection of a foreign gas and thermal gradients giving rise to large density variations. These developments are being carried out and the results will be presented in a future article.

In the present formulation, the simplifying assumption that the density viscosity ratio $\bar{\lambda}$ is unity has been adopted. In addition, the following features are noted: (a) the application of Euler transformation to equation (28), (b) the use of the local Dahmkohler number $\varphi(\xi)$ instead of ξ as the independent variable, and (c) the truncation procedure leading to the simple result in equation (50). It seems that all these features contribute to the simplicity and accuracy of the present method. It should be interesting to explore the general problem of nonsimilar solutions of binary diffusion in boundary layers by extending the present approach.

REFERENCES

1. D. MEKSYN, *New Methods in Laminar Boundary Layer Theory*. Pergamon Press, Oxford (1961).
2. G. R. INGER, Dissociated laminar boundary-layer flows over surfaces with arbitrary distributions of catalycity, *Int. J. Heat Mass Transfer* **6**, 815–822 (1963).
3. P. L. CHAMBRÉ and A. ACRIVOS, On chemical surface reactions in laminar boundary-layer flows, *J. Appl. Phys.* **27** (11), 1323–1328 (November 1956).

4. P. M. Chung and A. D. Anderson, Heat Transfer to Surfaces of Finite Catalytic Activity in Frozen Dissociated Hypersonic Flow, NASA TN D-350 (January 1961).
5. P. M. Chung, S. W. Liu and H. Mirels, Effects of discontinuity of surface catalycity on boundary-layer flow of dissociated gas, *Int. J. Heat Mass Transfer* **6**, 193–210 (1963).
6. T. Y. Li and P. S. Kirk, An approximate analytical derivation of skin friction and heat transfer in laminar binary boundary-layer flow, *Int. J. Heat Mass Transfer* **8**, 1217–1234 (1965).
7. P. G. Simpkins, On the diffusion of species in similar boundary layers with continuously varying properties, *Int. J. Heat Mass Transfer* **8**, 99–110 (1965).
8. N. C. Freeman, The diffusion of species in frozen hypersonic boundary layers, in *Fundamental Phenomena in Hypersonic Flow*, pp. 133–153. Cornell University Press (1966).

Résumé—On étend la méthode de Meksyn [1] pour la résolution des couches limites laminaires afin de traiter un problème typique de couche limite binaire avec une solution qui n'est pas en similitude. On s'occupe ici de la méthode asymptotique de résolution de l'écoulement de couche limite laminaire dissociée figée sur une plaque plane avec une catalyticité répartie de façon arbitraire. Une solution analytique est obtenue et on l'a comparé avec les données existantes dues à d'autres chercheurs [2–5]. La comparaison montre que les résultats actuels ont une précision convenable. L'attrait de la méthode employée réside dans la simplicité de la théorie.

Zusammenfassung—Die Methode von Meksyn [1] zur Lösung laminarer Grenzschichtprobleme wird zur Behandlung typischer Probleme binärer Grenzschichten mit nichtähnlichen Lösungen erweitert. Die vorliegende Arbeit umfasst die asymptotische Methode der Lösung einer eingefrorenen dissoziierten laminaren Grenzschichtströmung an einer ebenen Platte mit beliebig verteilter Katalysität. Eine geschlossene Lösung liess sich erhalten und wurde mit Ergebnissen anderer Autoren [2–5] verglichen. Dieser Vergleich zeigt, dass die heir angegebenen Resultate angemessene Genauigkeit besitzen. Der Vorteil der gezeigten Methode liegt in der Einfachheit der Analysis.

Аннотация—Метод расчета ламинарного пограничного слоя Мексина (1) обобщается на случай неавтомодельного решения типичной задачи бинарного пограничного слоя. Эта статья посвящена асимптотическому методу решения замороженного диссоциированного течения в пограничном слое на поверхности плоской пластины с произвольно распределенной каталитичностью. Решение получено в замкнутой форме, и результаты сравниваются с данными других исследователей (2–5). Сравнение показало удовлетворительную точность результатов. Преимущество данного метода состоит в его простоте.